GENERAL

probability and statistics solved problems

Published: September 3, 2026 | Index ID: #-

Introduction

Probability and statistics are the cornerstones of modern data analysis, from predicting stock market trends to designing experiments in medical research. Yet, many students and professionals find themselves tangled in formulas, unsure how to apply concepts to real-world scenarios. This article offers a comprehensive, step by step guide that tackles a wide range of **probability and statistics solved problems**. By following clear explanations, illustrative examples, and practical tips, you'll gain confidence in both the theory and its application. Whether you're preparing for exams, working on research projects, or simply curious about data science, the strategies outlined here will help you master the art of problem solving in probability and statistics.

Why Mastering Solved Problems Matters

Engaging with solved problems provides several key benefits:

- Conceptual Clarity: Seeing a problem worked out step by step reinforces the underlying principles.
- Pattern Recognition: Repeated exposure reveals common structures and shortcuts.
- Confidence Building: Knowing how to tackle diverse questions reduces anxiety during exams or project deadlines.
- Skill Transfer: Techniques learned in one context often apply to seemingly unrelated problems.

Core Topics Covered in Probability and Statistics

Below is an overview of the main areas we'll explore, each illustrated with solved examples:

1. Basic Probability – Events, Sample Spaces, and Rules
2. Conditional Probability and Bayes' Theorem
3. Random Variables – Discrete and Continuous
4. Probability Distributions – Binomial, Poisson, Normal, etc.
5. Expectation, Variance, and Standard Deviation
6. Sampling Theory and Estimation
7. Hypothesis Testing – t tests, chi square, ANOVA
8. Regression Analysis and Correlation
9. Non Parametric Methods
10. Resampling Techniques – Bootstrap and Jackknife

Step by Step Problem Solving Framework

Approaching a probability or statistics problem methodically reduces errors and improves efficiency. Follow this framework for consistent results:

1. Read Carefully: Identify what is given, what is required, and any constraints.
2. Define Variables: Assign symbols to unknowns and known quantities.
3. Choose the Right Tool: Decide whether to use a formula, theorem, or simulation.
4. Set Up the Equation: Translate the problem into mathematical language.

5. Compute: Carry out calculations step by step, checking units and logic.
6. Interpret: Convert the numerical answer back into the problem's context.
7. Verify: Cross check with alternative methods or sanity checks.

Example: Simple Probability Problem

Suppose a fair six sided die is rolled twice. What is the probability of getting a sum of 7?

Solution:

1. Define the event: \\(E = \\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\\}\\).
2. Count favorable outcomes: 6.
3. Total outcomes: \\(6 \\times 6 = 36\\).
4. Probability: \\(P(E) = \\frac{6}{36} = \\frac{1}{6}\\).

This simple example illustrates how to translate a verbal description into a mathematical solution.

In Depth Solved Problems by Category

1. Basic Probability

- Problem: A bag contains 4 red, 3 blue, and 2 green marbles. If two marbles are drawn without replacement, what is the probability that both are red?
- Solution:

  First draw: \\(P(R_1) = \\frac{4}{9}\\).
- Second draw: \\(P(R_2|R_1) = \\frac{3}{8}\\).
- Joint probability: \\(P(R_1 \\cap R_2) = \\frac{4}{9} \\times \\frac{3}{8} = \\frac{1}{6}\\).

2. Conditional Probability & Bayes' Theorem

- Problem: A test for a disease has a 95% sensitivity and a 90% specificity. In a population where 1% have the disease, what is the probability that a person who tests positive actually has it?
- Solution:

  Let \\(D\\) = disease, \\(T^+\\) = positive test.
- Compute \\(P(D|T^+) = \\frac{P(T^+|D)P(D)}{P(T^+)}\\).
- Where \\(P(T^+) = P(T^+|D)P(D) + P(T^+|\\bar{D})P(\\bar{D})\\).
- Plugging values: \\(P(T^+|D)=0.95\\), \\(P(D)=0.01\\), \\(P(T^+|\\bar{D})=0.10\\), \\(P(\\bar{D})=0.99\\).
- Result: \\(P(D|T^+) \\approx 0.083\\) or 8.3%.

3. Random Variables & Distributions

- Problem: A factory produces light bulbs with lifespans that follow an exponential distribution with a mean of 800 hours. What is the probability that a randomly selected bulb lasts longer than 1200 hours?
- Solution:

  For an exponential distribution, \\(P(X > x) = e^{-x/\\lambda}\\), where \\(\\lambda\\) is the mean.
- Here, \\(\\lambda = 800\\).
- Compute: \\(P(X > 1200) = e^{-1200/800} = e^{-1.5} \\approx 0.2231\\).

4. Expectation and Variance

- Problem: A fair coin is flipped 10 times. Let \\(X\\) be the number of heads. Find \\(E[X]\\) and \\(Var(X)\\).
- Solution:

Since \\(X \\sim Binomial(n=10, p=0.5)\\).

- Expectation: \\(E[X] = np = 10 \\times 0.5 = 5\\).
- Variance: \\(Var(X) = np(1-p) = 10 \\times 0.5 \\times 0.5 = 2.5\\).

5. Sampling and Estimation

- Problem: A survey of 200 students finds that 120 prefer online learning. Estimate the proportion of the entire student body that prefers online learning and construct a 95% confidence interval.
- Solution:

Sample proportion: \\(\\hat{p} = \\frac{120}{200} = 0.6\\).

- Standard error: \\(SE = \\sqrt{\\frac{\\hat{p}(1-\\hat{p})}{n}} = \\sqrt{\\frac{0.6 \\times 0.4}{200}} \\approx 0.0346\\).
- Margin of error: \\(1.96 \\times SE \\approx 0.0678\\).
- Confidence interval: \\(0.6 \\pm 0.0678 = (0.5322, 0.6678)\\).

6. Hypothesis Testing – t Test

- Problem: A researcher claims that the average time to complete a task is 15 minutes. A sample of 25 participants yields a mean of 14.2 minutes with a standard deviation of 2.1 minutes. Test the claim at the 5% significance level.
- Solution:

Null hypothesis \\(H_0: \\mu = 15\\).

- Alternative \\(H_a: \\mu \\neq 15\\).
- Compute t statistic: \\(t = \\frac{\\bar{x} - \\mu_0}{s/\\sqrt{n}} = \\frac{14.2 - 15}{2.1/\\sqrt{25}} = \\frac{-0.8}{0.42} \\approx -1.905\\).
- Degrees of freedom: \\(df = 24\\).
- Critical t at 5% two tailed: \\(\\pm 2.064\\).
- Since \\(|t|

7. Chi Square Goodness of Fit

- Problem: A die is rolled 60 times, yielding counts: {9, 12, 10, 8, 11, 10}. Does the die appear fair at the 5% significance level?
- Solution:

Expected count per face: \\(E = 60/6 = 10\\).

- Chi square statistic: \\(\\chi^2 = \\sum \\frac{(O_i - E)^2}{E}\\). Compute each term:

\\((9-10)^2/10 = 0.1\\)

- \\((12-10)^2/10 = 0.4\\)
- \\((10-10)^2/10 = 0\\)
- \\((8-10)^2/10 = 0.4\\)
- \\((11-10)^2/10 = 0.1\\)
- \\((10-10)^2/10 = 0\\)

8. Simple Linear Regression

- Problem: A dataset contains hours studied and exam scores for 10 students: \\((2, 70), (4, 75), (6, 80), (8, 85), (10, 90), (12, 92), (14, 94), (16, 95), (18, 96), (20, 97)\\). Fit a regression line and predict the score for 22 hours.
- Solution:

Compute means: \\(\\bar{x} = 11\\), \\(\\bar{y} = 88.4\\).

- Compute slope \\(b = \\frac{\\sum (x_i - \\bar{x})(y_i - \\bar{y})}{\\sum (x_i - \\bar{x})^2} = 1.55\\).
- Intercept \\(a = \\bar{y} - b\\bar{x} = 88.4 - 1.55 \\times 11 = 72.45\\).
- Regression line: \\(y = 72.45 + 1.55x\\).

Baca Juga (Artikel Terkait)

- **automaker mini software version v8 erptoboss**
  URL: http://apexgalvanizing.com/automaker-mini-software-version-v8-erptoboss.pdf
- **automata and computability**
  URL: http://apexgalvanizing.com/automata-and-computability.pdf
- **automata and languages theory and applications**
  URL: http://apexgalvanizing.com/automata-and-languages-theory-and-applications.pdf
- **automata and mechanical toys**
  URL: http://apexgalvanizing.com/automata-and-mechanical-toys.pdf

Tags: #probability #statistics #solved #problems